Numerical Integration Excel Add-In

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Introduction

This document describes an Excel Add-In that allows the user to do a 1D numerical integration over a finite interval of the form:

$$\int_a^b f(x)dx$$

where *a* and *b* defines the range of the integral and must be finite real numbers and *f*(*x*) is real valued through out the range. It is assumed that the integral is not divergent.

Add-In Installation

To install the Add-In, first save the Numerical integration Add-In on the local machine. For Macintosh, one should save it in Applications/Microsoft Office 2004/Office/Adin-ins folder. For Windows, ideally one should save it in one of two places:

1. Documents and Settings/<user name>/Application Data/Microsoft/AddIns folder
2. Library folder or one of its subfolders in the Microsoft Office/Office folder

These may be invisible folders on your particular machine. Otherwise any place is fine, however once placed it should not be move.

Next start Excel, select Tools>Add-Ins… (Fig. 1).

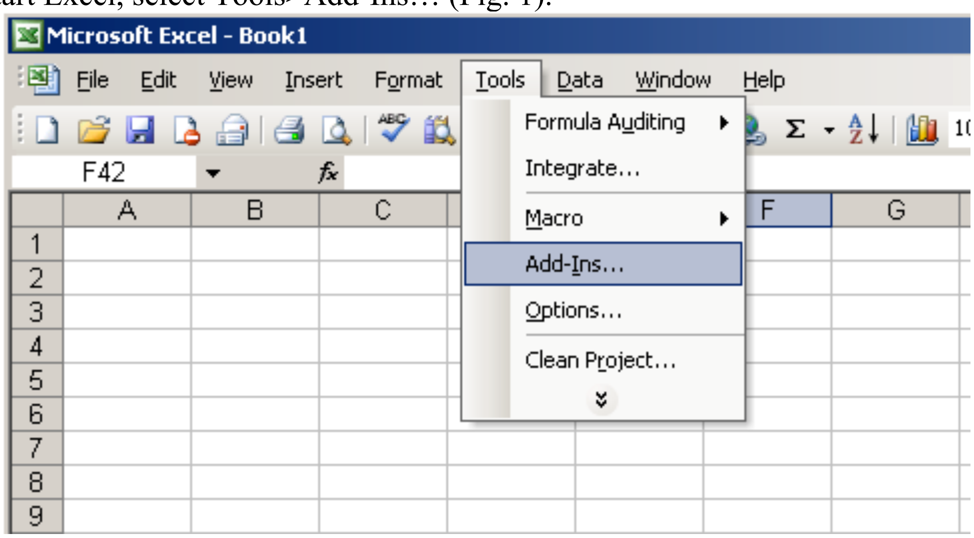

Fig. 1: Add-Ins… menu

This will display the Add-Ins Dialog box, which would look similar to Fig. 2.

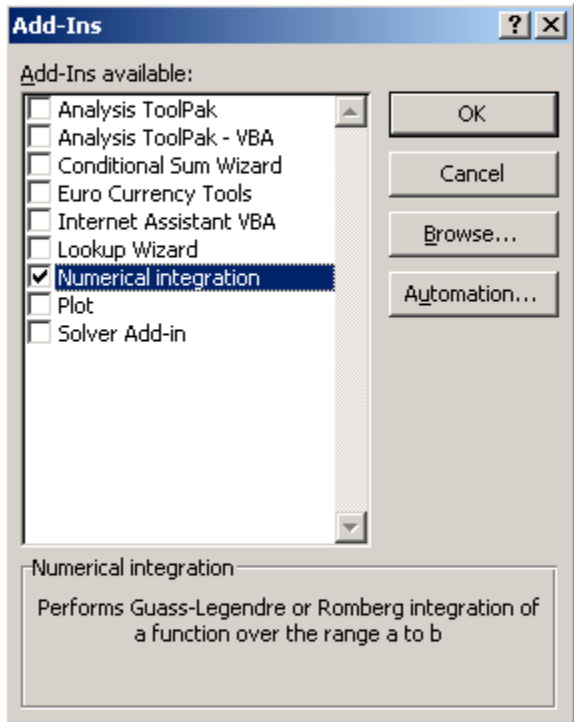

Fig. 2: Add-In dialog box

Select *Numerical integration* by checking the box. If *Numerical integration* is not shown, then press the Browse… button and browse to the Numerical integration.xla file you stored locally. Once selected, press OK.

Use

By selecting the Tools menu, you now have a new sub-menu named Integrate… (Fig. 3).

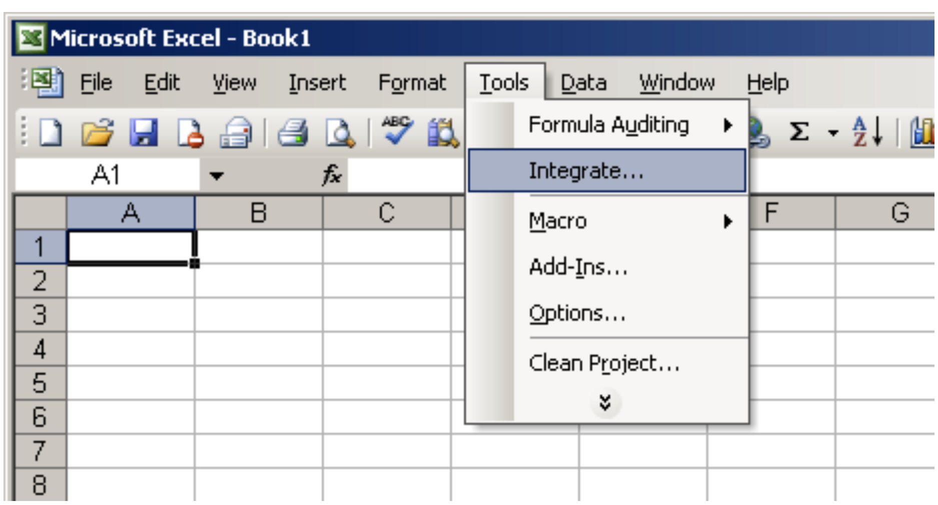

Fig. 3: New Integrate… sub-menu

Selecting the Integrate… sub-menu, you will be presented with the Integrate dialog box (Fig 4).

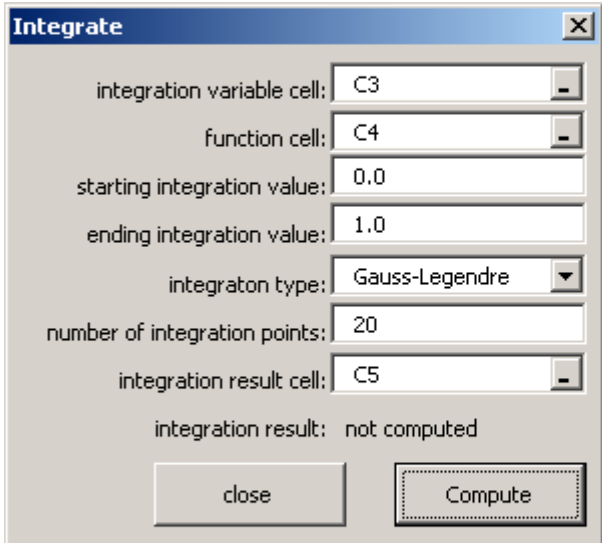

Fig. 4: Integrate dialog box

The *integration variable cell* box allows you to select with the mouse a the cell that holds the integration variable (*x* in our integration). The selection must be a single cell and must be on the current active sheet.

The *function cell* box allows you to select with the mouse a cell that holds the integrand (*f*(*x*) in our integration). The selection must be a single cell and must be on the current active sheet. The integrand must be real valued over the integration regime

The *starting integration value* box allows you to type in the starting value of the integral (*a* in our integration). The value must be a real number.

The *ending integration value* box allows you to type in the ending value of the integral (*b* in our integration). The value must be a real number.

The *integration type* pop-up allows for two different numerical quadratures: Gauss-Legendre and Romberg. Gauss-Legendre quadrature expands the integrand in terms of Legendre polynomials over the range of the integration. If the integrand is, in fact, a polynomial in the independent variable (*x*) of degree *n* then as long as the number of integration points (see below) is at least $(n+1)/2$, the integral will be exact. Romberg quadrature varies the number of integration points using a mid-point method and then tries to extrapolate the result of the integration in the limit of infinitesimal step sizes. As in all numerical quadratures, any general integral (even those that may exist analytically) may return incorrect answers.

The *number of integration points* box is only available for Gauss-Legendre quadratures. It is the number of integration points for the Gauss-Legendre quadrature and must be a natural number (i.e. positive integer).

The *integration result cell* box allows you to either type in the cell address or if you click the right side of the box, select the cell with the mouse that will hold the result of the integration. The selection may be a single cell, multiple cells or no cells at all. If a cell is selected it must be on the current active sheet.

The *Compute* button will try to compute the integral. This may be fast or take a few moments, depending on the type of quadrature used, the number of quadrature points requested and the speed of the computer. Eventually, the result of the integration will be returned in the cell(s) specified in the *integration result cell* and on the *integration result* line in the Integrate dialog box. If during the integration, an unrecoverable error occurs, the value of "Error" will be returned (Fig. 5). If Romberg quadrature was chosen and the extrapolation to infinitesimal step size failed to converged, a numerical result will be returned, however it will be highlighted in a font colored by red. Such results should be viewed with suspicion (Fig. 6).

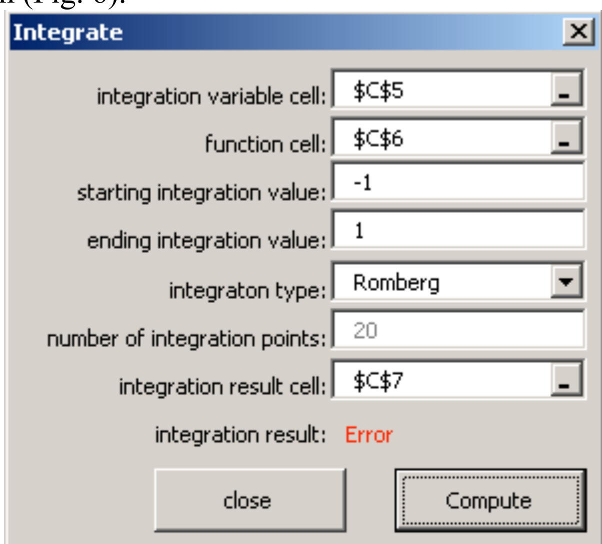

Fig. 5: Unrecoverable integration error

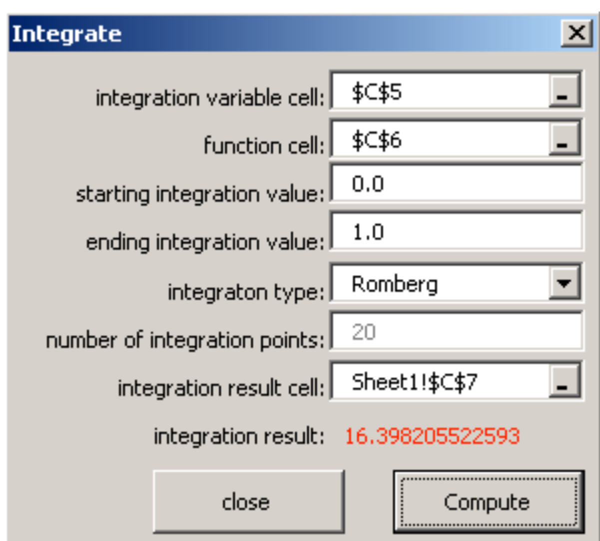

Fig. 6: Recoverable integration error

The *close* button will close the Integrate dialog box.

Hints and Tricks

Almost all numerical integration technique must assume that the integrand, $f(x)$, is 'smooth' (e.g. analytic). While it is possible to integrate functions that do not obey such conditions, one must be even more careful about evaluating if the resulting integral actually converged to the correct value.

The question that one should always ask from any numerical quadrature: is the result converged? For the Romberg quadrature, the result is most likely converged if the result returned did not suffer a recoverable integration error (Fig. 6). For Gauss-Legendre quadrature, the easiest way to test for convergence is to simply increase the number of integration points, perhaps by a factor of two. If the result changes, increase the number of points again. In either case, another way to test convergence is to split the integral in two. The sum of these two integrals must be the same as the original integral. This refinement can be done various times and may show convergence or show regions where the integrand is not behaving correctly.

Romberg integration tries to insure that the error in the integral is smaller than 10^{-5} of the absolute value of the integral. The actual error can actually be larger or smaller. Again, if the result is returned in a font colored red, it's mostly likely the integral failed to converge (either analytically or numerically).

Both numerical quadratures never sample the integrand precisely at the end points of the integral. This means that even if the integrand doesn't exist (i.e. singular) at either of these points and yet the integral does, one can expect and accurate result. For example, the integral

$$-\int_0^1 \ln(x)dx = x - x\ln(x)\Big|_0^1 = 1$$

has the integrand $\rightarrow -\infty$ at the low end of the integration. However, evaluating the integral one finds Romberg integration gives the result of 0.999987163611147. Gauss-Legendre gives 0.998496952500217 and 0.999984293999082 for 20 and 200 integration points respectively.

Let us do this integral. Load the Numerical integration.xla Add-In and open a new workbook. On a blank worksheet, reproduce Fig. 7. The labels ("x:", "f(x):", "result:") in column B are not necessary. The value in cell C3 for x can be arbitrary. Select the Integrate… sub-menu from the Tools menu and change the Integrate dialog box as shown in Fig. 8 (i.e. change *integration result cell* to point to C5). Notice that in this case, default cells for *integration variable cell* and *function cell* were correctly guessed. Change *integration type* and/or *number of integration points* and then press *Compute* to confirm the quoted values above. When done, press the *close* button.

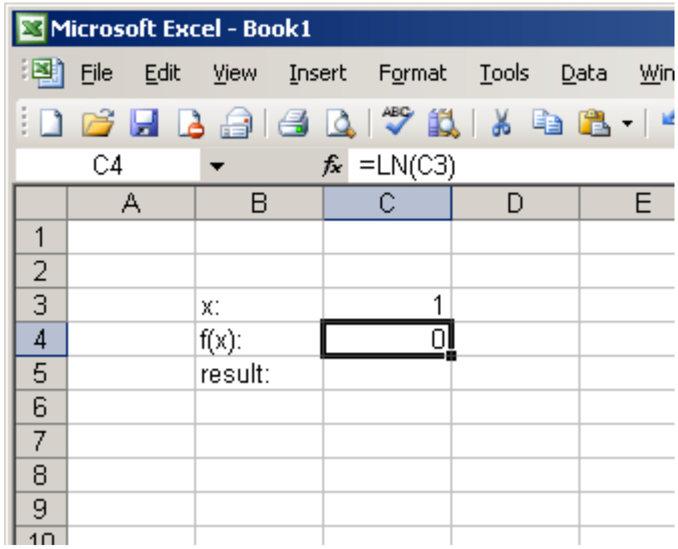

Fig. 7: Example worksheet

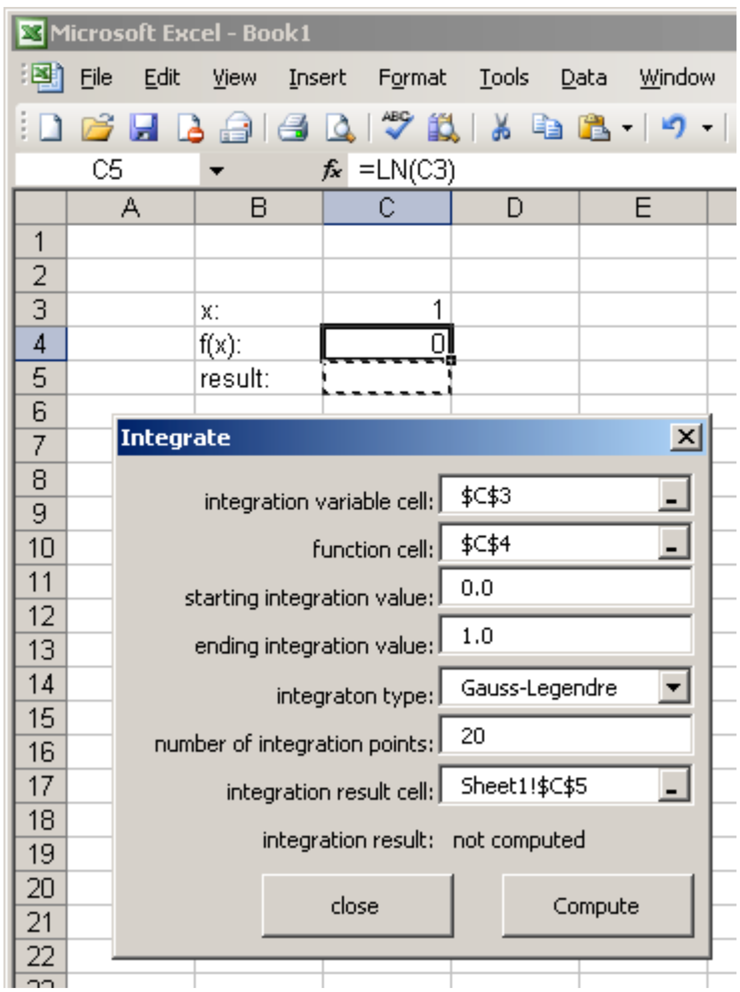

Fig. 8: Example Integrate dialog

Since the integrand is never evaluated the end points, one can always do the following transformation

$$\int_a^b f(x)dx = \int_{1/b}^{1/a} \frac{1}{t^2} f\left(\frac{1}{t}\right) dt \quad ab > 0$$

and can be used with either $b \to \infty$ and a positive **or** $a \to -\infty$ and b negative for any $f(x)$ that decreases toward infinity faster than $1/x^2$.

Similarly, if the integrand diverges as $(x-a)^{-\gamma}$, $0 \le \gamma < 1$, near $x = a$, then

$$\int_a^b f(x)dx = \frac{1}{1-\gamma} \int_0^{(b-a)^{1-\gamma}} t^{\frac{\gamma}{1-\gamma}} f\left(t^{\frac{1}{1-\gamma}} + a\right) dt \quad b > a.$$

Whereas if the integrand diverges at the upper limit, then

$$\int_a^b f(x)dx = \frac{1}{1-\gamma} \int_0^{(b-a)^{1-\gamma}} t^{\frac{\gamma}{1-\gamma}} f\left(b - t^{\frac{1}{1-\gamma}}\right) dt \quad b > a.$$

Of course, if the integrand is singular in this way at certain points within the integration range, then the integral can be split up at these points and then each integral can be done separately and then added together to get the final result.

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Contact

Like everyone else these days, I have a website at www.parker9.com and the page concerning this program is at www.parker9.com/script.html. Furthermore, email can be sent to NI@parker9.com.